## CMP 338: Third Class

HW 2 solution
Conversion between bases
The TINY processor
Abstraction and separation of concerns
Circuit design big picture
Moore's law and chip fabrication cost
Performance
What does to measure?
Processor performance and execution time The CPUtime equation

For next class: HW 3; review 1.6; read 1.10, 2.1-2.3

## HW 2: Basic Processor Model

In two or three sentences of your own words define, describe, or discuss the following components of the basic processor model:

1. Register
2. Register File
3. $\mathbb{A}(B)$ multiplexer (MUX)
4. A (B) Bus
5. ALU - Arithmetic / Logical Unit
6. C Bus
7. Program Counter (PC)
8. Instruction Register (IR)

## Basic Computer Model



## Basic Processor Model



## HW 2 : Processor Components

## Register

Memory containing one word (32 bits) or one double word ( 64 bits) of data.

## Register File

A collection of (usually 32) consecutively numbered (0 .. 31) registers.
$\triangle$ (B) multiplexer (MUX)
Selects a specified register to feed to the A (B) bus.

## HW 2 : Processor Components

A (B) Bus
Takes the value (or its complement) from the register determined by its MUX and copies it to one of the inputs of the ALU.

ALU — Arithmetic / Logical Unit
Performs an arithmetic (+, -, etc.) operation or a logical (bitwise and, or, etc.) operation of the values given it on the $A$ and $B$ buses and puts the result on the $C$ bus.

## HW 2 : Processor Components

C Bus
Can take a value from the ALU and place it in a specified register. Also, an extension of the System bus onto the processor. Used to fetch instructions into the IR, load a specified register with the value at a specified address in main memory, and store the value in a specified register into a specified address in main memory.

## HW 2 : Processor Components

Program Counter (PC)
Instruction Address Register contains the address in main memory of the next instruction to be executed.

## Instruction Register (IR)

Contains the instruction currently being executed. Determines what operations are performed by other components of the processor during the execution.

## Basic Processor Model



## The TINY Computer



## Integers in Different Bases

Base 10 (decimal - ten fingers)

$$
4129_{10}=4^{*} 10^{3}+1^{*} 10^{2}+2^{*} 10^{1}+9^{*} 10^{0}
$$

Base 2
(binary - two fingers)

$$
1011_{2}=1^{*} 2^{3}+0^{*} 2^{2}+1^{*} 2^{1}+1^{*} 2^{0}
$$

Base 16 (hexadecimal - sixteen fingers)

$$
A 3 F 8_{16}=10^{*} 16^{3}+3^{*} 16^{2}+15^{*} 16^{1}+8^{*} 16^{0}
$$

Conversion between base 2 and base 16 is easy!

$$
\text { 0x A3F8 = Ob } 1010001111110100
$$

## Four Hundred and Thirty Seven

| $437{ }_{10}$ | $110110101_{2}$ | 1B5 $_{16}$ |
| :---: | :---: | :---: |
| $4 \cdot 10^{2}=100$ | $1 \cdot 2^{8}=256$ | $1 \cdot 16^{2}=256$ |
| $+3 \cdot 10^{1}=30$ | $+1 \cdot 2^{7}=128$ | $+11 \cdot 16^{1}=128$ |
| $+7 \cdot 10^{\circ}=7$ | $+0 \cdot 2^{6}=0$ | $+5 \cdot 16^{0}=5$ |
|  | $+1 \cdot 2^{5}=32$ |  |
|  | $+1 \cdot 2^{4}=16$ |  |
|  | $+0 \cdot 2^{3}=0$ |  |
|  | $+1 \cdot 2^{2}=4$ |  |
|  | $+0 \cdot 2^{1}=0$ |  |
|  | $+1 \cdot 2^{0}=1$ |  |

## Base $2 \leftrightarrow$ Base 10

From base 2 to base 10
Add the power of 2 corresponding to each 1
Example: $01100100_{2}=2^{6}+2^{5}+2^{2}=64+32+4=100_{10}$
From base 10 to base 2
Express number as sum of distinct powers of 2

$$
209_{10}=128+64+16+1=1 \cdot 2^{7}+1 \cdot 2^{6}+1 \cdot 2^{4}+1 \cdot 2^{0}
$$

Add zero times the missing powers of 2

$$
209_{10}=1 \cdot 2^{7}+1 \cdot 2^{6}+0 \cdot 2^{5}+1 \cdot 2^{4}+0 \cdot 2^{3}+0 \cdot 2^{2}+0 \cdot 2^{1}+1 \cdot 2^{0}
$$

Write coefficients from highest to lowest power of 2 $209_{10}=11010001_{2}$

## Powers of 2

$\begin{array}{llll}2^{0} & 1 & 2^{8} & 256\end{array}$
$2^{1}$
2
$2^{9}$
512
$2^{2}$
4
$2^{10}$
1024
$2^{3}$
8
$2^{11}$
2048
$2^{4}$
16
$2^{12} 4096$
$2^{5}$
32
$2^{13}$
8192
$2^{6}$
64
$2^{14} 16384$
$2^{7}$
128
$2^{15} \quad 32968$

## Base $16 \leftrightarrow$ Base 2

From base 16 to base 2
Replace each hex digit with its 4-bit binary equivalent $6 E 30 A C 58_{16}=01101110001100001010110001011000_{2}$
From base 2 to base 16
Pad left with 0 until length is multiple of 4 $11001001001111011_{2}=00011001001001111011_{2}$
Replace consecutive sequences of 4 bits with hex digit $00011001001001111011_{2}=1927 B_{16}$

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |

## First 16 Non Negative Integers

Decimal

$$
0 \quad 8
$$

$$
19
$$

210
311
4
5
6
7
715

Binary
00001000
00011001
00101010
00111011
01001100
01011101
01101110
01111111
7
F

# 8 Great Computer Architecture Ideas 

Design for Moore's Law
Use abstraction to simplify design
Make the common case fast
Performance via parallelism
Performance via pipelining
Performance via prediction
Hierarchy of memories
Dependability via redundancy

## Separation of Concerns

## Interface

Boundary - between objects or systems
Protocol - rules for interaction between parties
Contract - formalized expectations
Distribution of Labor
User (consumer) ignores implementation
Provider (producer) ignores application
Instruction Set Architecture ISA
Between hardware and software
Application Program Interface API
Between application program and operating system

## Interface Map

## API

ISA $\longrightarrow$

What Happens to Your Program

## Computer Design - The Big Picture

A computer is one big sequential circuit
Abstract into discrete sequential components
Combinational circuits + memory + clock
Combinational circuit design

1. Specify semantics

Black Box input and output
Truth Table (Input determines output)
2. Truth table $\rightarrow$ Boolean formula
3. Minimize boolean formula (Karnaugh Maps)
4. Boolean formula $\rightarrow$ combinational circuit

## Synchronous Sequential Circuit



## The TINY Computer



## Black Box Logic Design

## Combinational circuit

Output determined by input
Design process

1. Specify semantics

Black Box input and output
Truth Table (input determines output)
2. From truth table to boolean formula
3. Minimize boolean formula (optional)

Boolean algebra
Karnaugh maps
4. From boolean formula to circuit

## Two Way Multiplexer Design



Informal semantics:

$$
\begin{aligned}
& \mathbf{X}=\mathbf{A}-\text { if } \mathbf{S}=0 \\
& \mathbf{X}=\mathbf{B}-\text { if } \mathbf{S}=1
\end{aligned}
$$

## Two Way Multiplexer Truth Table

> S A B X
> $0 \quad 0 \quad 0 \quad 0$
> $0 \begin{array}{llll}0 & 1 & 0\end{array}$
> $\begin{array}{llll}0 & 1 & 0 & 1\end{array}$
> not $S$ and $A$ and not $B$, or not $S$ and $A$ and $B$, or $S$ and not $A$ and $B$, or $S$ and $A$ and $B$
> $X=\bar{S} A \bar{B}+\bar{S} A B+S \bar{A} B+S A B$
> 1111
> $=\overline{\mathrm{S}} \mathrm{A}+\mathrm{SB}$

## Two Way Multiplexer Circuit



$$
X=\bar{S} A+S B
$$

8 Great Computer Architecture Ideas
Design for Moore's Law
Use abstraction to simplify design
Make the common case fast
Performance via parallelism
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Performance via prediction
Hierarchy of memories
Dependability via redundancy

## Moore’s "Law"

Moore's Observation (1965)
\# gates per chip doubles (about) every two years
Compute power $\propto$ \# gates per chip

What to do with increasing compute power?
Until about 2000, faster (and bigger) uniprocessors Since 2003, more (simpler) processors per chip

Exploiting increasing parallelism isn't easy
Are we the end of Moore's Law?
Seems to be slowing down, can't continue forever However, it has been pronounced dead before

Moore's Law

Processor Performance over Time

## Alpern's Law

## Exponential growth is ultimately unsustainable



## Alpern's Law

## Exponential growth is ultimately unsustainable



## Chip Fabrication



## Intel Core I7 Wafer



## Integrated Circuit Fabriction Costs

$$
\begin{aligned}
\text { Cost per die } & =\frac{\text { Cost per wafer }}{\text { Dies per wafer } X \text { yield }} \\
\text { Dies per wafer } & =\frac{\text { Wafer area }}{\text { Die area }} \\
\text { Yield } & =\frac{1}{\left(1+(\text { Defects per area } \times \text { Die area } / 2)^{2}\right.}
\end{aligned}
$$

11.8 inch (300mm) patterned wafer
$\sim 325$ (20.7 x 10.5 mm ) dies per wafer
$\sim 23 \%$ of dies are defective (yield $=\sim 0.77$ )
If each wafer costs $\$ 20,000$
what is the fabrication cost of a chip (die)?

## Understanding Performance

From qualitative to quantitative analysis
Statistical tools
Average and weighted average
Performance equations
Relative performance
CPU time equation
Amdahl's law
Performance metrics (what to measure)
What does "performance" mean?

## Performance Metrics

Different measures of airplane "performance"?
Speed (mph) ?
Range (miles) ?
Capacity (passengers) ?
Throughput (passengers miles per hour) ?

| Alplane | Passenger <br> capacity | Grilaing range <br> (milos) | Grulsing speed <br> (m.ph.) | Passenger throughput <br> (passengers $\times$ m-p.h.) |
| :--- | :---: | :---: | :---: | :---: |
| Boeing 777 | 375 | 4630 | 610 | 228,750 |
| Boeing 747 | 470 | 4150 | 610 | 286,700 |
| BAC/Sud Concorde | 132 | 4000 | 1350 | 178,200 |
| Douglas DC-8-50 | 146 | 8720 | 544 | 79,424 |

## Airplane Performance Metrics






## Computer Performance Metrics

Response (execution) time (seconds)

$$
\mathbf{C P U}
$$

Throughput (tasks per hour)
Availability (percent) $\frac{M T T F}{M T T F+M T T R}$
MTTF — Mean Time To Failure (years)
MTTR - Mean Time To Repair (minutes)
Execution energy (joules)
Throughput cost (tasks per hour per dollar)

## Response Time Performance

Definition
Performance $_{X} \equiv \frac{1}{\text { ExecutionTime }_{X}} \quad \mathrm{P}_{X} \equiv \frac{1}{\mathrm{E}_{X}}$
Better performance mean shorter execution time
Relative performance
X is $\boldsymbol{n}$ times as fast as Y if and only if

$$
\boldsymbol{n}=\frac{\mathrm{P}_{X}}{\mathrm{P}_{Y}}=\frac{\mathrm{E}_{Y}}{\mathrm{E}_{X}}
$$

Y takes $\boldsymbol{n}$ times as long as X to execute

## Relative Performance

If computer A runs a program in 10 seconds and computer B runs the same program in 15 seconds, how much faster is A than $B$ ?

We know that A is $n$ times as fast as B if

$$
\frac{\text { Performance }_{\mathrm{A}}}{\text { Performance }_{\mathrm{B}}}=\frac{\text { Execution time }_{\mathrm{B}}}{\text { Execution time }_{\mathrm{A}}}=n
$$

Thus the performance ratio is

$$
\frac{15}{10}=1.5
$$

and A is therefore 1.5 times as fast as B .
In the above example, we could also say that computer B is 1.5 times slower than computer A, since

$$
\frac{\text { Performance }_{\mathrm{A}}}{\text { Performance }_{\mathrm{B}}}=1.5
$$

means that

$$
\frac{\text { Performance }_{\mathrm{A}}}{1.5}=\text { Performance }_{\mathrm{B}}
$$

## Processor Performance

Program execution time $=\mathrm{CPU}_{\text {time }}+\mathrm{I} / 0_{\text {time }}$
$\mathrm{CPU}_{\text {time }}$ will be our key metric of processor performance We will return to $I / \mathrm{O}_{\text {time }}$ at the end of this course
$\mathrm{CPU}_{\text {time }}=\#$ instructions • (average) instruction time instruction $_{\text {time }}=($ average $)$ cycles per instruction $\cdot$ cycle $_{\text {time }}$ cycle $_{\text {time }}=\frac{\# \text { seconds }}{\text { cycle }}=\frac{1}{\text { clock }_{\text {rate }}}$
clock $_{\text {rate }}$ measured in Hertz (cycles per second)

$$
\mathrm{CPU}_{\text {time }}(\text { execution })=\frac{\# \text { instructions }}{\text { execution }} \cdot \frac{\# \text { cycles }}{\text { instruction }} \cdot \frac{\# \text { seconds }}{\text { cycle }}
$$

| Components of performance | Units of measure |
| :--- | :--- |
| CPU execution time for a program | Seconds for the program |
| Instruction count | Instructions executed for the program |
| Clock cycles per instruction (CPI) | Average number of clock cycles per instruction |
| Clock cycle time | Seconds per clock cycle |

Figure 1.15 shows the basic measurements at different levels in the computer and what is being measured in each case. We can see how these factors are combined to yield execution time measured in seconds per program:
Time $=$ Seconds $/$ Program $=\frac{\text { Instructions }}{\text { Program }} \times \frac{\text { Clock cycles }}{\text { Instruction }} \times \frac{\text { Seconds }}{\text { Clock cycle }}$
Always bear in mind that the only complete and reliable measure of computer performance is time. For example, changing the instruction set to lower the instruction count may lead to an organization with a slower clock cycle time or higher CPI that offsets the improvement in instruction count. Similarly, because CPI depends on type of instructions executed, the code that executes the fewest number of instructions may not be the fastest.

## Performance Equations

## Definition of performance

$$
\text { performance: } P_{x} \equiv \frac{1}{T_{x}} \quad \text { relative performance: } \frac{P_{x}}{P_{y}}=\frac{T_{y}}{T_{x}}
$$

CPU time equation

$$
T_{\text {CPU }}(\text { execution })=\frac{\# \text { instructions }}{\text { execution }} \cdot \frac{\# \text { cycles }}{\text { instruction }} \cdot \frac{\# \text { seconds }}{\text { cycle }}
$$

Amdahl's law

$$
T_{\text {new }}=\frac{\text { fraction effected } \cdot T_{\text {old }}}{\text { improvement }}+\text { fraction not effected } \cdot T_{\text {old }}
$$

## $\mathrm{CPU}_{\text {time }}$ Relative Performance

$$
T_{C P U}(\text { execution })=\frac{\# \text { instructions }}{\text { execution }} \cdot \frac{\# \text { cycles }}{\text { instruction }} \cdot \frac{\# \text { seconds }}{\text { cycle }}
$$

$T_{X}=\#$ instructions $_{X} \cdot C P I_{X} \cdot$ cycleTime $_{X}$
$=\frac{\# \text { instructions }_{X} \cdot C P I_{X}}{\text { clockRate }_{X}}$

$$
\text { cycleTime }=\frac{1}{\text { clockRate }}
$$

$$
\begin{aligned}
\frac{P_{X}}{P_{Y}}=\frac{T_{Y}}{T_{X}} & =\frac{\# \text { instructions }_{Y} \cdot C P I_{Y} \cdot \text { cycleTime }_{Y}}{\# \text { instructions }_{X} \cdot C P I_{X} \cdot \text { cycleTime }_{X}} \\
& =\frac{\# \text { instructions }_{Y} \cdot C P I_{Y} \cdot \text { clockRate }_{X}}{\# \text { instructions }_{X} \cdot C P I_{X} \cdot \text { clockRate }_{Y}}
\end{aligned}
$$

